

# Adiabatic passage for quantum gates in mesoscopic atomic ensembles

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We present schemes for geometric phase compensation in adiabatic passage which can be used for implementation of quantum logic gates with atomic ensembles consisting of an arbitrary number of strongly interacting atoms. Protocols using double sequences of stimulated Raman adiabatic passage (STIRAP) or adiabatic rapid passage (ARP) pulses are analyzed. Switching the sign of the detuning between two STIRAP sequences, or inverting the phase between two ARP pulses, provides state transfer with well defined amplitude and phase independent of atom number in the Rydberg blockade regime. Using these pulse sequences we present protocols for universal single-qubit and two-qubit operations in atomic ensembles containing an unknown number of atoms.

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Quantum information can be stored in collective states of ensembles of strongly interacting atoms [1]. This idea can be extended to encoding an entire register of qubits in ensembles of atoms with multiple ground states [2] which opens up the possibility of large quantum registers in a single atomic ensemble [3], or of coupling arrays of small ensembles in a scalable atom chip based architecture [4]. Quantum information based on ensembles can be realized more generally in any ensemble of strongly coupled spins [5]. Our proposal for implementing high fidelity quantum gates in ensembles is thus of interest for several different implementations of quantum computing.

The use of ensembles for qubit encoding is of interest for several reasons. The enhanced coupling to the radiation field by a factor of  $\sqrt{N}$ , with  $N$  the number of atoms or spins, is useful for coupling matter qubits to single photons [6]. Combining photon coupling with local quantum gates in ensembles enables architectures with improved fidelity for quantum networking [7]. The use of ensemble qubits is also attractive for deterministic loading of registers of single atom qubits [8] and for realizing gates that act on multiple particles [9],[10]. All of these capabilities rely on high fidelity quantum gate operations between collectively encoded qubits. However, due to the increase of the Rabi frequency of oscillations between different collective states proportional to  $\sqrt{N}$ ,  $\Omega_N = \sqrt{N}\Omega$ , with  $\Omega$  the one atom Rabi frequency, it is difficult to perform gates with well defined rotation angles in the situation where  $N$  is unknown [11, 12]. Although there is recent progress in nondestructive measurement of  $N$  with high accuracy [13] it remains an outstanding challenge to implement high fidelity quantum logic gates without precise knowledge of  $N$ , particularly in the case of collectively encoded registers [2] where the effective value of

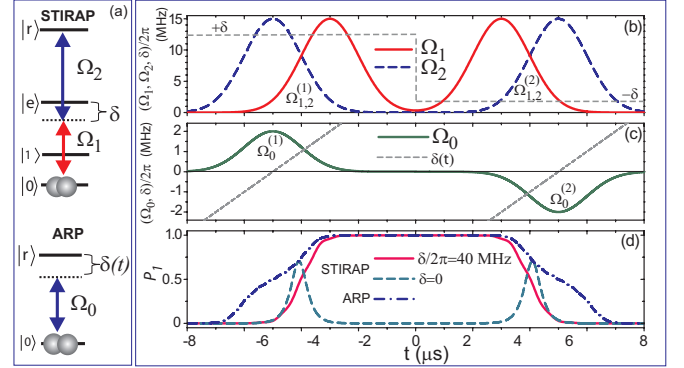


FIG. 1. (Color online). (a) Energy levels for two-photon STIRAP and single-photon ARP excitation; (b) Time sequence of STIRAP laser pulses; (c) Time sequence for ARP laser excitation; (d) Time dependence of the single-atom excitation probability.

$N$  depends on the unknown quantum state encountered during a computation.

Adiabatic passage techniques have been widely used for deterministic population transfer in atomic and molecular systems [14, 15]. These techniques have been studied for quantum state control [16], qubit rotations [17], creation of entangled states [18], and for deterministic excitation of Rydberg atoms [19, 20]. Although STIRAP or ARP methods provide pulse areas with strongly suppressed sensitivity to the Rabi frequency  $\Omega_N$ , and therefore suppressed sensitivity to  $N$ , the phase of the final state is in general still strongly dependent on  $N$ . Increase of the average atom number  $\bar{N}$  reduces the relative fluctuations by a factor of  $\sqrt{\bar{N}}$ , which is still not enough for scalable quantum computation with moderate size ensembles. Indeed gate errors at

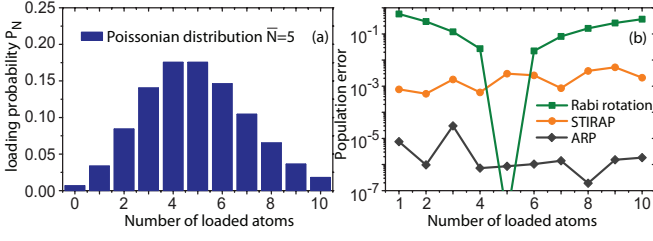


FIG. 2. (Color online). (a) Poissonian statistics of atom loading into an optical trap for  $\bar{N} = 5$ ; (b) Comparison of the fidelity of single-atom excitation by a  $\pi$  laser pulse with the area optimized for  $N = 5$  atoms ( $t = \pi/\sqrt{5}\Omega$ ), with a STIRAP sequence, and an ARP pulse. The single STIRAP sequence used  $\Omega_j(t) = \Omega_j e^{-(t+t_j)^2/2\tau^2}$  for  $j = 1, 2$  with  $\Omega_1/2\pi = 30$  MHz,  $\Omega_2/2\pi = 40$  MHz,  $t_1 = 3.5 \mu\text{s}$ ,  $t_2 = 5.5 \mu\text{s}$ ,  $\tau = 1 \mu\text{s}$ , and  $\delta/2\pi = 200$  MHz. The single ARP pulse used  $\Omega_0(t) = \Omega_0 e^{-t^2/2\tau^2}$  with  $\Omega_0/2\pi = 2$  MHz,  $\tau = 1 \mu\text{s}$ , and linear chirp  $\alpha/2\pi = (1/2\pi)(d\delta(t)/dt) = 1 \text{ MHz}/\mu\text{s}$  [20].

the level of 0.001 would require  $\bar{N} \sim 4000$  (accounting for fluctuations of  $\pm 2.5\sqrt{\bar{N}}$ ), and achieving full blockade for such a large ensemble remains an outstanding challenge. In this Letter we propose double adiabatic sequences using either STIRAP or ARP excitation which remove the phase sensitivity, and can be used to implement gates on collectively encoded qubits without precise knowledge of  $N$ .

Our approach is shown in Fig. 1. A sequence of two STIRAP pulses is produced with fields having Rabi frequencies  $\Omega_1$ ,  $\Omega_2$ , and detuning  $\delta$  from the intermediate state. A Rydberg state  $|r\rangle$  is excited by a counterintuitive sequence of laser pulses in the regime of Rydberg blockade, where excitation of more than one Rydberg atom is suppressed due to strong dipole-dipole interaction between the atoms [1]. The second reverse STIRAP sequence, as shown in Fig. 1(a), returns the Rydberg atom back to the ground state. The time dependence of the probability of single-atom Rydberg excitation in an ensemble of two interacting atoms is shown in Fig. 1(d) for  $\Omega_1 = \Omega_2 = 2\pi \times 15$  MHz and two different values of detuning. As we have shown in our recent work [20], large detuning from the intermediate excited state  $|e\rangle$  is required for deterministic single-atom excitation after the first STIRAP sequence, which is represented by the solid curve in Fig. 1(d). In the case of zero detuning, a single STIRAP sequence does not leave any population in the Rydberg state [broken curve in Fig. 1(d)], in agreement with the calculations of Ref. [18]. Deterministic single-atom excitation can also be achieved using linearly chirped ARP pulses, as shown in Figs. 1(c),(d).

The probability of loading  $N$  atoms in a small optical or magnetic trap is described, in general, by Poissonian statistics. Figure 2(a) shows the Poissonian distribution for  $\bar{N} = 5$ . For  $\bar{N} = 5$  the probability to load zero atoms is 0.0067, which is small enough to create a large quantum register with a small number of defects [21].

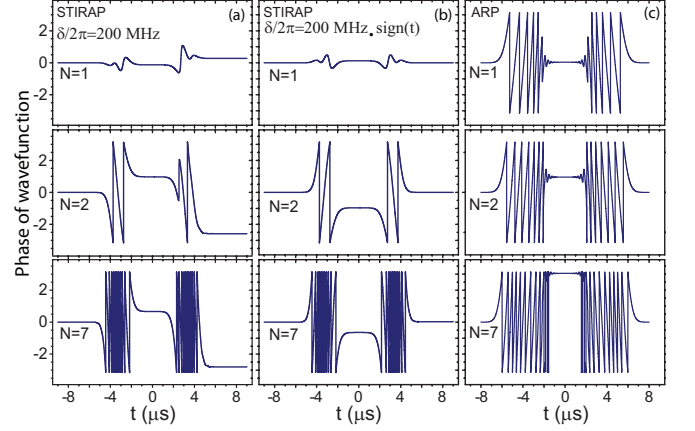


FIG. 3. (Color online). Calculated time dependence of the phase of the collective ground state amplitude for  $N = 1, 2, 7$  atoms (top to bottom). Double STIRAP sequence with  $\delta/2\pi = 200$  MHz (a), with  $\delta/2\pi = 200 \text{ MHz} \times \text{sign}(t)$  (b), and for a double ARP pulse sequence with phase inversion (c). All other parameters are as in Fig. 2.

Figure 2(b) shows a comparison of the fidelity of single-atom excitation for a single-photon  $\pi$  rotation with the area optimized for  $N = 5$  atoms compared to STIRAP or ARP pulses. We see that the adiabatic pulses reduce the population error by up to several orders of magnitude for a wide range of  $N$ . Calculations were performed using the Schrödinger equation, neglecting spontaneous emission, and assuming perfect blockade so only states with at most a single Rydberg excitation were included.

At the end of a double STIRAP sequence the population is returned back to the collective ground state  $|000\dots\rangle$  of the atomic ensemble, but a geometric phase is accumulated. This phase shift of the ground state is dependent on the Rabi frequency and leads to gate errors. The phase of the atomic wavefunction can be compensated by switching the sign of the detuning between two STIRAP sequences, or by switching the phase between two ARP pulses, as shown in Fig. 3. For a double STIRAP sequence with the same detuning throughout the accumulated phase depends on  $N$  [Fig. 3(a)], while the phase change is zero, independent of  $N$ , when we switch the sign of detuning  $\delta$  between the two STIRAP sequences [Fig. 3(b)]. A similar phase cancellation occurs for  $\pi$  phase shifted ARP pulses [Fig. 3(c)] [22]. Although the proposed double-pulse sequences are almost insensitive to variations of the Rabi frequency, they are not immune to errors caused by fluctuations of the Rabi frequencies between the first and second pulses. The dependence of the population and phase errors on parameters of the laser pulses are shown in Fig. 4. The population transfer error in the ensemble of  $N = 5$  atoms can be kept below  $10^{-3}$  for STIRAP [Fig. 4(a)] and  $10^{-4}$  for an ARP pulse [Fig. 4(b)] for a wide range of Rabi frequencies. The dependence of the phase error on the ratio of Rabi frequencies  $\Omega_1^{(2)}/\Omega_1^{(1)}$  between pulses [see Fig. 1(b)] is shown

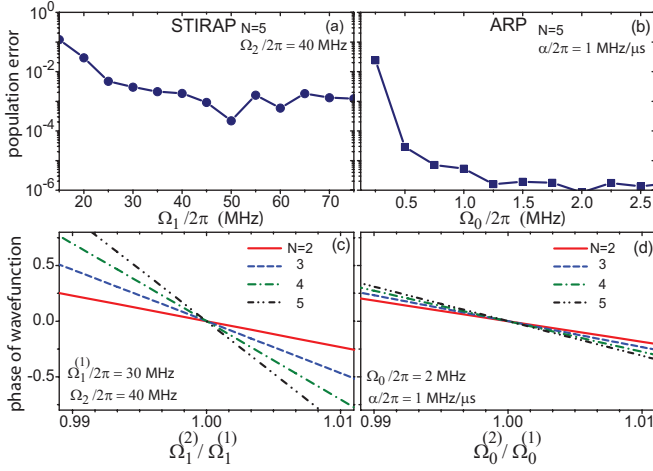


FIG. 4. (Color online). Dependence of the population error on the Rabi frequency with  $N = 5$  atoms for STIRAP (a) or ARP pulses (b). Dependence of the phase error on Rabi frequency changes between pulses for STIRAP (c) or ARP pulses (d).

in Fig. 4(c) for  $N = 1..5$  atoms. The ARP excitation in Fig. 4(d) demonstrates less sensitivity to fluctuations of the Rabi frequency. For either approach the double pulse amplitudes must be well matched for low phase errors. Due to the fact that the pulses are separated in time by only a few  $\mu\text{s}$  amplitude matching at the level of  $10^{-6}$  is feasible [23].

The phase compensated double STIRAP or ARP sequences can be used to implement a universal set of quantum gates as we now describe. Consider atoms with levels  $|0\rangle, |1\rangle, |e\rangle|r\rangle$  as shown in Fig. 1. A qubit can be encoded in an  $N$  atom ensemble with the logical states  $|\bar{0}\rangle = |000\dots 000\rangle$ ,  $|\bar{1}\rangle' = \frac{1}{\sqrt{N}} \sum_{j=1}^N |000\dots 1_j\dots 000\rangle$ . Levels  $|0\rangle, |1\rangle$  are atomic hyperfine ground states with coupling between these states and implementation of quantum gates mediated by the singly excited Rydberg state  $|\bar{r}\rangle' = \frac{1}{\sqrt{N}} \sum_{j=1}^N |000\dots r_j\dots 000\rangle$ . Rydberg blockade only allows single excitation of  $|r\rangle$  so the states  $|\bar{0}\rangle$  and  $|\bar{r}\rangle'$  experience a collectively enhanced coupling rate  $\Omega_N = \sqrt{N}\Omega$ . States  $|\bar{r}\rangle'$  and  $|\bar{1}\rangle'$  are coupled at the single atom rate  $\Omega$ . State  $|\bar{1}\rangle'$  is produced by the sequential application of  $\pi$  pulses  $|\bar{0}\rangle \rightarrow |\bar{r}\rangle'$  and  $|\bar{r}\rangle' \rightarrow |\bar{1}\rangle'$ .

Pulse areas independent of  $N$  on the  $|0\rangle \leftrightarrow |r\rangle$  transition can be implemented with STIRAP or ARP as described above. We will define the logical basis states and the auxiliary Rydberg state as  $|\bar{0}\rangle = |000\dots 000\rangle$ ,  $|\bar{1}\rangle = e^{i\chi_N}|\bar{1}\rangle'$ , and  $|\bar{r}\rangle = e^{i\chi_N}|\bar{r}\rangle'$ . Here  $\chi_N$  is the phase produced by a single  $N$  atom STIRAP pulse with positive detuning. We assume that we do not know the value of  $N$ , which may vary from qubit to qubit, and therefore  $\chi_N$  is also unknown, but has a definite value for fixed  $N$ .

For universal quantum computing we need a complete set of gates which can be comprised of single qubit rotations and a two-qubit entangling gate. Arbitrary single

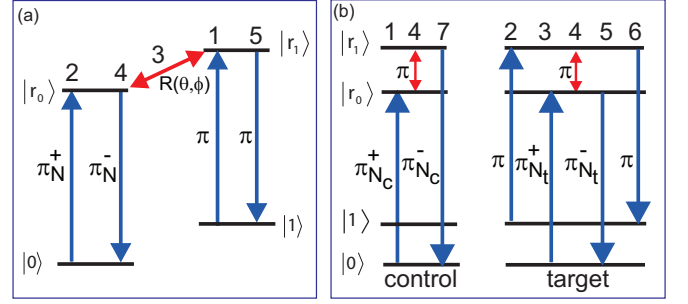


FIG. 5. (color online) (a) Single qubit gate for a mesoscopic qubit with  $N$  atoms. Pulses 1 – 5 act between the qubit states  $|0\rangle, |1\rangle$  and the Rydberg states  $|r_0\rangle, |r_1\rangle$ . Pulses 1, 2, 4, 5 are optical transitions and pulse 3 is a microwave frequency transition between Rydberg states. (b) CNOT gate between mesoscopic qubits with  $N_c$  atoms in the control qubit and  $N_t$  atoms in the target qubit.

qubit rotations in the basis  $|\bar{0}\rangle, |\bar{1}\rangle$  can be performed with high fidelity, without precise knowledge of  $N$ , by accessing several Rydberg levels as shown in Fig. 5(a). Starting with a qubit state  $|\psi\rangle = a|\bar{0}\rangle + b|\bar{1}\rangle$  we perform a sequence of pulses 1-5, giving the sequence of states

$$\begin{aligned} |\psi_1\rangle &= a|\bar{0}\rangle + ib|\bar{r}_1\rangle \\ |\psi_2\rangle &= a|\bar{r}_0\rangle + ib|\bar{r}_1\rangle \\ |\psi_3\rangle &= a'|\bar{r}_0\rangle - ib'|\bar{r}_1\rangle \\ |\psi_4\rangle &= a'|\bar{0}\rangle - ib'|\bar{r}_1\rangle \\ |\psi_5\rangle &= a'|\bar{0}\rangle + b'|\bar{1}\rangle. \end{aligned} \quad (1)$$

The final state  $|\psi\rangle = a'|\bar{0}\rangle + b'|\bar{1}\rangle$  is arbitrary and is selected by the rotation  $R(\theta, \phi)$ , in step 3:  $\begin{pmatrix} a' \\ -b' \end{pmatrix} = \mathbf{R}(\theta, \phi) \begin{pmatrix} a \\ b \end{pmatrix}$ . Depending on the choice of implementation, to be discussed below, this may be a one- or two-photon microwave pulse. Provided states  $|r_0\rangle, |r_1\rangle$  are strongly interacting, and limit the entire ensemble to a single Rydberg excitation in either state, the indicated sequence is obtained. In the regime of  $\Omega_3$  large compared to the Rydberg excitation rates the time spent populating a Rydberg level corresponds to  $4\pi$  of Rydberg pulse area. This is the same as for a single atom  $C_Z$  gate, and we therefore expect the limit on gate infidelity to be  $\sim 0.002$  [24] for small ensembles. For moderate size ensembles the gate error is expected to increase linearly with  $N$  [10]. The five pulse sequence we describe here is more complicated than the three pulses needed for an arbitrary single qubit gate in the approach of Ref. [1]. The reason for this added complexity is that the special phase preserving property of the double STIRAP or ARP sequences requires that all population is initially in one of the states connected by the pulses. The sequence of pulses in Fig. 5(a) ensures that this condition is always satisfied.

A CNOT gate can be implemented in several ways.

The standard construction [25] of  $H(t) - C_Z - H(t)$  can be used where the Hadamard gates are performed as in Fig. 5(a), and the  $C_Z$  operation is implemented in the same way as for single atom qubits [26] using  $\pi_{|\bar{1}\rangle-|\bar{r}\rangle}(c) \ 2\pi_{|\bar{1}\rangle-|\bar{r}\rangle}(t) \ \pi_{|\bar{1}\rangle-|\bar{r}\rangle}(c)$ , where  $c(t)$  stand for control(target) qubits. The CNOT gate therefore requires a total pulse area of  $12\pi$  Rydberg pulses. We can reduce this to  $7\pi$  of Rydberg pulses as shown in Fig. 5(b) which implements an approach analogous to the amplitude-swap gate demonstrated for single atom qubits in [27]. Starting with an arbitrary two-qubit state  $|\psi\rangle = a|\bar{0}\bar{0}\rangle + b|\bar{0}\bar{1}\rangle + c|\bar{1}\bar{0}\rangle + d|\bar{1}\bar{1}\rangle$  we generate the sequence of states

$$\begin{aligned} |\psi_1\rangle &= a|\bar{r}_0\bar{0}\rangle + b|\bar{r}_0\bar{1}\rangle + c|\bar{1}\bar{0}\rangle + d|\bar{1}\bar{1}\rangle \\ |\psi_2\rangle &= a|\bar{r}_0\bar{0}\rangle + b|\bar{r}_0\bar{1}\rangle + c|\bar{1}\bar{0}\rangle + id|\bar{1}\bar{r}_1\rangle \\ |\psi_3\rangle &= a|\bar{r}_0\bar{0}\rangle + b|\bar{r}_0\bar{1}\rangle + c|\bar{1}\bar{r}_0\rangle + id|\bar{1}\bar{r}_1\rangle \\ |\psi_4\rangle &= ia|\bar{r}_1\bar{0}\rangle + ib|\bar{r}_1\bar{1}\rangle + ic|\bar{1}\bar{r}_1\rangle - d|\bar{1}\bar{r}_0\rangle \\ |\psi_5\rangle &= ia|\bar{r}_1\bar{0}\rangle + ib|\bar{r}_1\bar{1}\rangle + ic|\bar{1}\bar{r}_1\rangle - d|\bar{1}\bar{0}\rangle \\ |\psi_6\rangle &= ia|\bar{r}_1\bar{0}\rangle + ib|\bar{r}_1\bar{1}\rangle - c|\bar{1}\bar{1}\rangle - d|\bar{1}\bar{0}\rangle \\ |\psi_7\rangle &= ia|\bar{0}\bar{0}\rangle + ib|\bar{0}\bar{1}\rangle - c|\bar{1}\bar{1}\rangle - d|\bar{1}\bar{0}\rangle. \end{aligned} \quad (2)$$

The gate matrix is therefore

$$U_{\text{CNOT}} = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

which can be converted into a standard CNOT gate with a single qubit rotation. All pulses except 4 are optical and are localized to either the control or target qubit. Pulse 4 is a microwave field and drives a  $\pi$  rotation on both qubits. As for the single qubit gate the requirement for correct operation is that the interactions  $|r_0\rangle \leftrightarrow |r_0\rangle$ ,  $|r_1\rangle \leftrightarrow |r_1\rangle$ ,  $|r_0\rangle \leftrightarrow |r_1\rangle$  all lead to full blockade of the ensembles. Since the frequency of pulse 4, which is determined by the energy separation of states  $|r_0\rangle, |r_1\rangle$ , can be chosen to be very different from the qubit frequency given by the energy separation of states  $|0\rangle, |1\rangle$  the application of microwave pulses will not lead to crosstalk in an array of ensemble qubits.

We now discuss the feasibility of implementation of these gate protocols in a Rydberg blockaded ensemble. To get an isotropic interaction suitable for ensemble blockade we use  $ns$  states [28]. The long range interaction strength can be parameterized with a  $C_6$  coefficient as  $V(n, n') = C_6^{(n, n')}/R^6$  with  $R$  the atomic separation. For Cs  $ns$  states the optimum gate fidelity is obtained for  $80s$  [24], and the interaction strengths for  $|r_0\rangle = |80s_{1/2}, m = 1/2\rangle, |r_1\rangle = |81s_{1/2}, m = 1/2\rangle$  are  $C_6^{(80,80)} = 3.2$ ,  $C_6^{(80,81)} = 5.1$ ,  $C_6^{(81,81)} = 3.7$ , in units of  $10^6 \text{ MHz } \mu\text{m}^6$ . Rydberg  $ns$  states can be accessed starting from a ground  $s$  state using two-photon STIRAP pulses. These  $ns$  states can also be excited with two-photon ARP pulses where one photon is fixed frequency

and one is chirped. Alternatively single photon ARP pulses can be used to access  $np$  states. Although the interaction of  $np$  states is not isotropic it can be made isotropic in lower dimensional 1- or 2-D lattices by orienting the quantization axis perpendicular to the lattice symmetry plane. For Cs the optimal state is  $112p_{3/2}$  [24], and the interaction strengths for  $|r_0\rangle = |112p_{3/2}, m = 3/2\rangle, |r_1\rangle = |113p_{3/2}, m = 3/2\rangle$  at 90 deg. to the quantization axis are  $C_6^{(112,112)} = 250.$ ,  $C_6^{(112,113)} = 820.$ ,  $C_6^{(113,113)} = 270.$ , in units of  $10^6 \text{ MHz } \mu\text{m}^6$ . We see that for both  $ns$  and  $np$  states a strong interaction is obtained for all involved Rydberg states as desired. The pulse connecting  $|r_0\rangle, |r_1\rangle$  can be implemented as a 2-photon electric dipole transition at microwave frequencies via a neighboring opposite parity state. The large transition dipoles of Rydberg states scaling as  $n^2ea_0$  ( $e$  is the electronic charge,  $a_0$  is the Bohr radius) render fast microwave pulses straightforward to implement. At  $n = 80$ , a detuning of 1 GHz from the intermediate state, and a very modest  $1 \mu\text{W}/\text{cm}^2$  microwave power level, gives  $\sim 25 \text{ MHz}$  two-photon Rabi frequency.

The requirements for gates that are insensitive to the value of  $N$  are within reach of current experimental capabilities. Optimization of the Rabi frequency and a rigorous determination of the gate fidelity could be found following the methods of Ref. [24]. These collective gates require more Rydberg pulses than single atom gates which will result in different choices for the Rabi frequencies and optimal quantum numbers  $n$  compared to the values used for illustration above which are optimal for single atom qubits. The influence of a finite strength Rydberg interaction must also be accounted for. Such a calculation is beyond the scope of this Letter and will be presented elsewhere. The collective gates do require that  $N$  does not change during the operation of the gate. In any realistic implementation with trapped atoms gate times are significantly shorter than atom loss times, so this is not an issue. On longer time scales atom loss leads to qubit errors at a physical level, as is also true for a single atom encoding. Ideas for correcting physical errors due to atom loss in ensembles have been presented in Ref. [29].

In summary we have analyzed double STIRAP and ARP sequences with phase compensation for quantum gates in collectively encoded ensembles. Our analysis shows that high fidelity universal gates can be achieved using available experimental resources. We have made explicit calculations for  $N = 1..10$  showing the performance of these pulse schemes. For larger ensembles, with smaller fractional variation in  $N$ , the performance will be even better. We anticipate that these ideas will contribute to realization of quantum logic using collectively encoded qubits and registers.

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